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SOME PROBLEMS OF SORPTION AND DESORPTION DYNAMICS IN BIPOROUS MEDIA†

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Solutions of problems of mass transfer in a semi-infinite biporous medium in which convective transfer of a liquid or gas occurs in the macropores and molecular diffusion and sorption (desorption) in the transverse direction takes place in the micropores of sorbent (ionite) grains are presented. Solutions of analogous axially symmetric problems with impulsive and constant supply of the substance being absorbed into a cylindrical cavity are also obtained.

1. FORMULATION OF THE PROBLEMS

THE FOLLOWING assumptions are made when formulating problems of sorption dynamics. The macropores in the sorbent (ionite) layer form a system of channels, along which the motion of a liquid or gas occurs. The walls (grains) forming the macropores have inner porosity δ due to meso- and micropores, which can be penetrated by the molecules (ions) of the absorbed substance. If the kinetic (external) relative surface S of the particles forming the macropores and the kinetic porosity m, that is, the ratio of the volume of macropores to the total sorbent volume, are known, then the structural model can be represented as a system of parallel plates with the following characteristic dimensions: the half-thickness $a=(1-m)/S$ of a wall and the half-thickness $-b_0=m/S$ of a channel (gap).

For a semi-infinite medium it is assumed that the y coordinate axis is perpendicular to the surface of the medium. Convective transfer of the substance being absorbed (or carrier) occurs inside the macropores with velocity v parallel to the y direction. In the case of sorption, molecular (ionic) transfer of the substance into the pores of the layer a occurs in the x direction, or, in the case of desorption, in the opposite direction. In a real biporous medium the flow splits into a set of separate streams moving past the sorbent grains along curvilinear trajectories intersecting one another. As a result, the solution (gas) is mixed up, and its concentration in the macropores (in the gap b_0) can be assumed to be constant at any distance y. The case when the substance being absorbed accumulates at the surface is also considered.

For the axially symmetric problem it is also assumed that convective radial mass transfer of a gas or liquid occurs along the macropores, while molecular (ionic) transfer of the substance into the sorbent grains occurs in the transverse direction along circular arcs of radius r. If the walls (grains) are small, then the arcs can be replaced by the chords $0\leqslant x\leqslant(a+b_0)$. This enables one to reduce the problem to the one-dimensional case, namely, the transfer of a substance from the plate (gap) b_0 into the layer a. Radial diffusion inside the grains is ignored. Because of convection, the concentration of the substance in the layer b_0 is also taken to be constant for any r. On the upper and lower boundaries of the infinite sorbent layer (stratum) of thickness R the axial flows of the liquid or gas are equal to zero. The carrier or the substance

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being absorbed comes from a cylindrical cavity (a perfect hole) of radius R and height h at a constant rate Q. The r coordinate is measured relative to the axis of the cylindrical cavity.

Four problems are considered. In Problem 1 it is assumed that, at $y=0$, a flow of substance of concentration c_{02} is supplied at a constant rate into a semi-infinite medium with initial concentration c_{01}. In Problem 2 the carrier transfers the substance being absorbed with initial concentration c_0 from deeper layers to the surface, where it is removed (evaporated), while the non-volatile substance being absorbed accumulates at the surface.

In Problem 3 it is assumed that the substance being absorbed is supplied into a cylindrical cavity at $t=0$ and spreads immediately, which makes it possible to obtain uniform initial concentration c_0 of the substance in the whole volume $V=\pi R^2 h$ of the cavity. In Problem 4 the substance being absorbed is supplied into the cavity at a rate Q with constant concentration c_{02}. The initial concentration of the substance in the sorbent is c_{01}.

The equation of mass transfer and absorption of substances for a semi-infinite medium can be written as follows:

for the layer b_0

$$\frac{\partial c}{\partial t} = D_y \frac{\partial^2 c}{\partial y^2} \mp v \frac{\partial c}{\partial y} - \frac{\partial c^*}{\partial t} \tag{1.1}$$

(the minus sign corresponds to Problem 1, while the plus sign corresponds to Problem 2);

for the layer a

$$\frac{\partial u}{\partial t} = D_x \frac{\partial^2 u}{\partial x^2} - \frac{\partial u^*}{\partial t}, \quad \frac{\partial u^*}{\partial t} = k_1 u - k_2 u^* \tag{1.2}$$

Here c, c^*, u, u^* are, respectively, the concentrations of the substance being absorbed in the macropores (the layer b_0), entering the grains of the sorbent, inside the micropores in the sorbent grains (the layer a) and absorbed by the sorbent (ionite), D_y and D_x are the coefficients of the longitudinal convective and transverse molecular (ionic) diffusion and k_1 and k_2 are the constants of the equation of sorption kinetics.

System (1.1), (1.2) is solved under the following initial and boundary conditions.

For Problem 1

$$t=0; \quad c=u=c_{01} \tag{1.3}$$

$$x=a, \quad c=u, \quad b_0 \partial c^*/\partial t = \delta D_x \partial u/\partial x \tag{1.4}$$

$$t=0, \quad \partial u/\partial t = 0; \ x=0, \quad \partial u/\partial x = 0 \tag{1.5}$$

$$y \to \infty, \quad \partial c/\partial y = 0 \tag{1.6}$$

$$y=0, \quad -D_y \partial c/\partial y + vc = vc_{02} \tag{1.7}$$

The same conditions are used for Problem 2, except that it is necessary to set $c_{01}=c_0$ and $c_{02}\equiv 0$ and replace v by $-v$ in (1.3) and (1.7).

For Problems 3 and 4, Eq. (1.1) is replaced by

$$\frac{\partial c}{\partial t} = D_r \left(\frac{\partial^2 c}{\partial r^2} + \frac{1}{r}\frac{\partial c}{\partial r}\right) - \frac{v_0 R}{r}\frac{\partial c}{\partial r} - \frac{\partial c^*}{\partial t} \tag{1.8}$$

where D_r is the coefficient of radial convective diffusion, $v=v_0 R/r$, and $v_0=Q/(2\pi Rhm)$.

For a sorbent grain (the a layer), system (1.2), (1.8) is solved under the conditions (1.3)–(1.5), and, for the b_0 layer, under the conditions

$$r \to \infty, \quad \partial c/\partial r = 0 \tag{1.9}$$

For Problem 3 (the balance equation)

$$r = R, \quad -\pi R^2 h \partial c/\partial t = -2\pi R h m (D_r \partial c/\partial r - vc) \tag{1.10}$$

and, for Problem 4

$$r = R, \quad -D_r \partial c/\partial r + vc = vc_{02} \tag{1.11}$$

2. PROBLEM 1

Eliminating u^*, from (2.1) by the well-known method [1], we can write the equation of sorption dynamics in the form

$$\frac{\partial^2 u}{\partial t^2} + (k_1 + k_2)\frac{\partial u}{\partial t} = D_x \frac{\partial^3 u}{\partial t \partial x^2} + k_2 D_x \frac{\partial^2 u}{\partial x^2} \tag{2.1}$$

In the domain of Laplace transforms (DLT) we obtain the equation

$$U_1'' - \alpha^2 U_1 = 0 \tag{2.2}$$

the solution of which, subject to conditions (1.3)–(1.5), is

$$U_1 = C_1 \frac{\mathrm{ch}\alpha x}{\mathrm{ch}\alpha a}, \quad \alpha = \left[\frac{p}{D_x}\left(1 + \frac{k_1}{p + k_2}\right)\right]^{1/2} \tag{2.3}$$

where U_1 and C_1 are the transforms of $u_1 = u - c_{01}$ and $c_1 = c - c_{01}$.

In the DLT Eq. (1.1) can be written in the form

$$C_1'' - \frac{v}{D_y} C_1' - \frac{p + \gamma}{D_y} C_1 = 0, \quad \gamma = \frac{D_x}{ab} \alpha a \,\mathrm{th}\alpha a \tag{2.4}$$

where γ is obtained by differentiating (2.3) with respect to x in accordance with the second condition in (1.4), and where $b = b_0/\delta$.

If conditions (1.6) and (1.7) and relations (2.3) are taken into account, Eq. (2.4) has the solution

$$U_1 = \frac{(c_{02} - c_{01}) v \,\mathrm{ch}\alpha x e^{-\beta_- y}}{D_y p \beta_+ \mathrm{ch}\alpha a}, \quad \beta_\pm = \left(\frac{v^2}{4D_y^2} + \frac{p + \gamma}{D_y}\right)^{1/2} \pm \frac{v}{2D_y} \tag{2.5}$$

We have $\mathrm{ch}\alpha a \neq 0$, since $\gamma \to \infty$ $(\beta \to \infty)$ for $\mathrm{ch}\alpha = 0$.

The inverse transform of (2.5) can be found using the Riemann–Mellin formula and integrating along a Hankel-type contour [2]. Since two roots p_j obtained from the equation

$$p^2 + (k_1 + k_2 - D_x \alpha^2) p - k_2 D_x \alpha^2 = 0 \tag{2.6}$$

correspond to one value $\alpha^2 = pD_x^{-1}[1 + k_1(p + k_2)^{-1}]$, integration along the cut edges with respect to $v = \alpha a$ and $\eta = \alpha a/i$ must be from 0 to ∞, as in the case of $|p|$.

We set

$$\mu_0 = -p \frac{D_x}{a^2}, \quad \mu_j = -p_j \frac{D_x}{a^2}, \quad K = \frac{k_1}{k_2}, \quad H = \frac{D_x}{k_2' a^2}, \quad A = \frac{a}{b} = \frac{a\delta}{b_0}$$

and, according to (2.6), we get

$$\mu_0 = [1 + K - \nu^2 H - \xi(\nu^2)]/(2H) > 0 \tag{2.7}$$

$$\mu_j = [1 + K + \eta^2 H - (-1)^j \xi(-\eta^2)]/(2H) > 0 \tag{2.8}$$

$$\xi(w) = [(1 + K^2)^2 - 2(K - 1)Hw + H^2 w^2]^{1/2}, \quad w = (\nu^2, -\eta^2)$$

We introduce new variables by setting

$$2\nu d\nu = -\kappa_0 d\mu_0, \quad \kappa_0 = 1 + K/(H\mu_0 - 1)^2$$

$$2\eta d\eta = \kappa_j d\mu_j, \quad \kappa_j = 1 + K/(1 - H\mu_j)^2$$

$$[1 + 4D_y v^{-2}(p + \gamma)]^{1/2} = \pm i\Delta_m \qquad (m = 0, 1, 2)$$

$$\Delta_0 = (\xi_0 - 1)^{1/2}, \quad \xi_0 = (\mu_0 - A\nu \mathrm{th}\nu)/N$$

$$\Delta_j = (\xi_j - 1)^{1/2}, \quad \xi_j = (\mu_j + A\eta \mathrm{tg}\eta)/N$$

$$N = \frac{(av)^2}{4D_x D_y} = \frac{z^2}{\epsilon}, \quad z = \frac{\mathrm{Pe}}{2}, \quad \mathrm{Pe} = \frac{vy}{D_y}, \quad Y = \frac{y}{a}, \quad \epsilon = Y^2 \frac{D_x}{D_y}$$

If $\xi_0 < 1$ and $\xi_j < 1$, then the values of the integrand are imaginary, and are therefore excluded when the integration is carried out.

Using the new variables, we can write down the solution in the form

$$\frac{u(X, Y, \mathrm{Fo}) - c_{01}}{c_{02} - c_{01}} = 1 - \frac{4}{\pi}\mathrm{Re}\left\{\sum_{j=1}^{2}\int_0^{\infty}\frac{\cos(\eta X)\phi_j \eta d\eta}{\cos\eta} - \int_0^{\infty}\frac{\mathrm{ch}(\nu X)\phi_0 \nu d\nu}{\mathrm{ch}\,\nu}\right\}$$

$$\Phi_m = [\sin(z\Delta_m) + \Delta_m \cos(z\Delta_m)]\exp(-\mu_m \mathrm{Fo} + z)/(\mu_m \xi_m \kappa_m) \tag{2.9}$$

$$X = \frac{x}{a}, \quad \mathrm{Fo} = \frac{D_x t}{a^2}$$

A computer program has been written to compute the integral (2.9). As an example, in Fig. 1 we show the initial sections of the graphs for $K = 200$, $H = 1000$, $\mathrm{Pe} = 2$, $N = 0.25$ and $A = 0.1$; 0.5; 0.9 (curves 1, 2, and 3, respectively). The values $\bar{c} = [c(y, t) - c_{01}](c_{02} - c_{01})$ become unity for $\mathrm{Fo} \approx 650$, 3130, 5620 for curves 1, 2, and 3, respectively.

The roots ν_0 and η_{jn} can be found from the characteristic equations $\Delta_0^2 = 0$ and $\Delta_j^2 = 0$. If $(K+1)/(NH) < 1$, then the second integral is equal to zero, since $\xi_0 < 1$ for all $\nu \geq 0$. If the roots are determined, the solution (2.9) can be rewritten as follows:

$$\frac{u(X, Y, \mathrm{Fo}) - c_{01}}{c_{02} - c_{01}} = 1 - \frac{4}{\pi}\left\{\sum_{j=1}^{2}\sum_{n=0}^{\infty}\int_{\eta_{jn}}^{(2n+1)\pi/2}\frac{\cos(\eta X)\Phi_j \eta d\eta}{\cos\eta} - \int_0^{\nu_0}\frac{\mathrm{ch}(\nu X)\Phi_0 \nu d\nu}{\mathrm{ch}\nu}\right\} \tag{2.10}$$

If $j = 1$ and $(K = 1)/(NH) < 1$, then the summation starts from $n = 0$, and if $(K+1)/(NH) > 1$, it starts from $n = 1$. If $c_{02} > c_{01}$, then a sorption process occurs, and if $c_{02} < c_{01}$, a desorption process occurs.

For $t \to \infty (p \to 0)$ we have $\mathrm{th}\alpha a \approx \alpha a$ and $X = 1$, and expression (2.5) can be simplified. Using transform tables [3], we can write the approximate solution as follows:

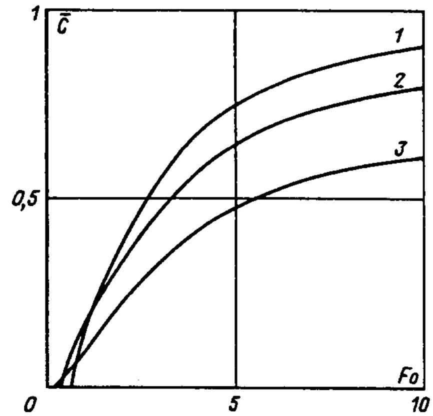

FIG. 1.

$$\frac{c(Y,\mathrm{Fo})-c_{01}}{c_{02}-c_{01}} \approx \frac{1}{2}\{\operatorname{erfc}S^{-} + \exp\mathrm{Pe}[2\sqrt{M\mathrm{Fo}^{*}}\, i\operatorname{erfc}S^{+} - \operatorname{erfc}S^{+}]\}$$

$$S^{\pm} = \frac{1}{2}\left(\frac{\mathrm{Pe}}{\sqrt{M\mathrm{Fo}^{*}}} \pm \sqrt{M\mathrm{Fo}^{*}}\right)$$

$$\mathrm{Fo}^{*} = \frac{\mathrm{Fo}}{1+A(1+K)}, \quad M = 4N = \frac{(av)^{2}}{D_{x}D_{y}} \tag{2.11}$$

3. PROBLEM 2

The solution in the DLT, like (2.5), can be obtained by setting $c_{01}=c_0$ and $c_{02}=0$ and replacing v by $-v$. Unlike (2.5), it has a double pole for $p=0$ and infinitely many roots, which can be found from the equation $p+\gamma=0$. On applying the theorem on residues and integrating along a Hankel-type contour, we can write the solution as follows:

$$\frac{u(X,Y,\mathrm{Fo})-c_0}{c_0} = \exp(-\mathrm{Pe})\left\{1-\mathrm{Pe}+\frac{M[\mathrm{Fo}-(1+K)(1-X^{2})/2]}{1+A(1+K)}+\right.$$
$$+\frac{MA[HK+(1+K)^{2}/3]}{[1+A(1+K)]^{2}} - M\frac{\operatorname{ch}(\theta_0 X)\exp(-\mu_0^{*}\mathrm{Fo})}{\operatorname{ch}\theta_0\mu_0^{*}\Psi_0} -$$
$$\left.- M\sum_{j=1}^{2}\sum_{n=1}^{\infty}\frac{\cos(\theta_{jn}X)\exp(-\mu_{jn}^{*}\mathrm{Fo})}{\cos\theta_{jn}\mu_{jn}^{*}\Psi_{jn}}\right\}+$$
$$+\frac{4}{\pi}\left\{\sum_{j=1}^{2}\sum_{n=0}^{\infty}\int_{\eta_{jn}}^{(2n+1)\pi/2}\frac{\cos(\eta X)\Phi_j^{*}\eta\, d\eta}{\cos\eta} - \int_{0}^{\nu_0}\frac{\operatorname{ch}(\eta X)\Phi_0^{*}\nu\, d\nu}{\operatorname{ch}\nu}\right\} \tag{3.1}$$

$$\Psi_0 = 1+\kappa_0^{*}[A+\mu_0^{*}/\theta_0^{2}-\mu_0^{*2}/(A\theta_0^{2})]/2$$
$$\Psi_{jn} = 1+\kappa_{jn}^{*}[A-\mu_{jn}^{*}/\theta_{jn}^{2}+\mu_{jn}^{*2}/(A\theta_{jn}^{2})]/2$$

The functions Φ_m^{*} are given by expressions (2.9) for Φ_m with z replaced by $-z$.

The roots θ_0 and θ_{jn} can be found from the characteristic equations $A\theta \mathrm{th}\theta - \mu_0^* = 0$ and $A\theta_j \mathrm{tg}\theta_j + \mu_j^* = 0$, respectively. The values μ_0^*, μ_{jn}^* correspond to expressions (2.7) and (2.8) with v and η replaced by θ_0 and θ_j. Moreover, $\kappa_0^* = 1 + K/(H\mu_0^* - 1)^2$ and $\kappa_{jn}^* = 1 + K/(1 - H\mu_{jn})^2$. The remaining remarks and notation connected with computing the integrals are given in Section 2.

For long times $t \to \infty$ it is possible to restrict ourselves to the initial terms of solution (3.1). We will also present another approximate solution. By analogy with (2.11), we have

$$\frac{c(Y, \mathrm{Fo}) - c_0}{c_0} \simeq \frac{1}{2}\{ \exp(-\mathrm{Pe})[2\sqrt{M\mathrm{Fo}^*}\ i\mathrm{erfc}S^- + \mathrm{erfc}S^-] - \mathrm{erfc}S^+ \} \tag{3.2}$$

Equation (3.2) simplifies when $y \to 0$ [4]

$$\frac{c(0, \mathrm{Fo}) - c_0}{c_0} \simeq 1 + M\mathrm{Fo}^* - 4i^2\mathrm{erfc}\left(\frac{1}{2}\sqrt{M\mathrm{Fo}^*} \right) \tag{3.3}$$

4. PROBLEM 3

In the DLT Eq. (1.8) takes the form [2]

$$\frac{d^2C}{dr^2} + \frac{1-2\nu}{r}\frac{dC}{dr} - \kappa^2 C = 0 \tag{4.1}$$

Taking (1.10) into account, the general solution of this equation can be expressed as

$$C = Ar^\nu K_\nu(\kappa r)$$
$$\kappa^2 = (p + \gamma)/D_r,\ \nu = v_0 R/(2D_r) = Q/(4\pi hmD_r) \tag{4.2}$$

($K_\nu(\kappa r)$ denotes the MacDonald function). In the DLT (1.10) can be rewritten as follows:

$$r = R,\ -pC + c_0 = -2mR^{-1}(D_r C' - vC) \tag{4.3}$$

After substituting (4.2) into the latter equality and some reduction, one can find the constant A. Taking (2.3) into account, the solution takes the final form

$$\frac{U(x, r, p)}{c_0} = \left(\frac{r}{R}\right)^\nu \frac{\mathrm{ch}\alpha x K_\nu(\kappa r)}{\mathrm{ch}\,\alpha a[g\kappa RK_{\nu+1}(\kappa R) + pK_\nu(\kappa R)]}$$
$$g = \frac{2mD_r}{R^2} \tag{4.4}$$

The original mapping (4.4) can be found by integrating along a Hankel-type contour [2] and has the form

$$\frac{u(X, Y, \mathrm{Fo})}{c_0} = \frac{2}{\pi}\rho^\nu \left\{ \sum_{j=1}^{2}\sum_{n=1}^{\infty} \int_{\eta_{jn}}^{(2n+1)\pi/2} \frac{\cos(\eta X)\Phi_j \eta d\eta}{\cos\eta} - \right.$$
$$\left. - \int_0^{\xi_0} \frac{\mathrm{ch}(\xi X)\overline{\Phi}_0 \xi d\xi}{\mathrm{ch}\xi} \right\} \tag{4.5}$$
$$\Phi_s = [\alpha_s Y_\nu(\Delta_s\rho) - \beta_s J_\nu(\Delta_s\rho)]\exp(-\mu_s \mathrm{Fo})/[\kappa_s(\alpha_s^2 + \beta_s^2)]$$
$$\alpha_s = \epsilon\Delta_s J_{\nu+1}(\Delta_s) - \mu_s J_\nu(\Delta_s)$$

$$\beta_s = \epsilon\Delta_s Y_{\nu+1}(\Delta_s) - \mu_s Y_\nu(\Delta_s)$$

$$\kappa_s = 1 + K/(1 - H\mu_s)^2, \quad (s = 0, 1, 2)$$

$$\Delta_0 = (\mu_0 - A\xi \mathrm{th}\xi)^{1/2}, \quad \Delta_j = (\mu_j + A\eta \mathrm{tg}\eta)^{1/2}$$

$$\rho = \frac{r}{R}, \quad \epsilon = 2m \frac{D_r}{D_x}\left(\frac{a}{R}\right)^2$$

where μ_0 is given by (2.7) with ν replaced by ξ, and where μ_j is given by (2.8). The roots ξ_0, η_{jn} $(n = 1, 2, 3, \ldots)$ can be found from the transcendental equations $\Delta_s^2 = 0$.

We shall obtain approximate solutions. Taking into account that $\kappa R \approx \sqrt{(p/D_r)}R$, $K_{\nu+1}(kR)/K_\nu(kR) \approx 1$ if $p \to \infty$ $(t \to 0)$ and $X = 1$, we write (4.4) for $r = R$ in the form

$$C(R, p)/c_0 \simeq (p + g\sqrt{pR/D_r})^{-1} \tag{4.6}$$

The function corresponding to this transform is given by

$$\frac{c(R, t)}{c_0} \approx e^{z^2} \mathrm{erfc}\, z \approx 1 - \frac{2}{\sqrt{\pi}} z, \quad z = \frac{2m\sqrt{D_r t}}{R} \tag{4.7}$$

Neglecting those terms of the series that contain powers of $p \to 0$ greater than one, we write (4.4) as follows:

$$U(x, y, p)/c_0 \approx [2\nu g B(p + 1/B)]^{-1}$$
$$B = \frac{a^2}{D_x} T, \quad T = \frac{1}{G} + \frac{m[1 + A(1 + K)][\nu(\rho^2 - 1) + 1]}{2\epsilon\nu(\nu - 1)} + \frac{(1 + K)(1 - X^2)}{2} \tag{4.8}$$
$$2\nu g = Q/V, \quad G = Qa^2/(VD_x), \quad V = \pi R^2 h, \quad X = x/a$$

which implies that

$$u(X, \rho, \mathrm{Fo})/c_0 \simeq (GT)^{-1}\exp(-\mathrm{Fo}/T) \quad (t \to \infty) \tag{4.9}$$

For $\rho = 1$ the second and third terms of the sum T can be omitted. Then

$$c(R, t)/c_0 \simeq \exp(-Qt/V) \tag{4.10}$$

Results of experiments [5] concerned with the variation of the concentration of indicator in the cavity under observation are presented in Fig. 2. The graph has three parts. The first part is connected with the initial flow, while, according to (4.9), the second part is connected with the subsequent flow of the indicator from the central cavity, into which it was supplied once at $t = 0$. The maximum indicator concentration in the cavity corresponds to the peak of the graph. The third part of the graph in Fig. 2 represents a slow flow of the indicator washed out (i.e. being desorbed) by the solvent flow in the region between the central and the closest neighbouring cavities.

If the solvent (water) is supplied at a constant rate Q into the central cavity, the indicator decreases exponentially, which is consistent with (4.10).

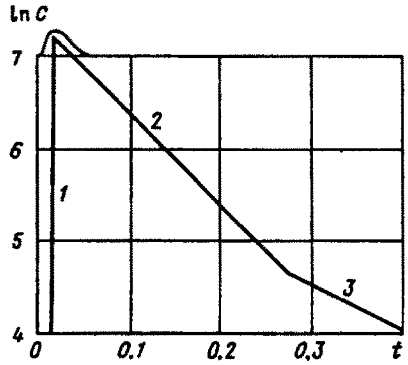

FIG. 2.

5. PROBLEM 4

Taking (1.11) into account, the solution in the DLT has the form

$$\frac{U_1(x,r,p)}{c_{02}-c_{01}} = 2\nu\left(\frac{r}{R}\right)^{\nu} \frac{\text{ch}\alpha x K_\nu(\kappa r)}{p\kappa R\,\text{ch}\alpha a K_{\nu+1}(\kappa R)} \tag{5.1}$$

The inverse transform can be found by integrating along a Hankel-type contour

$$\frac{u(X,\rho,\text{Fo})-c_{01}}{c_{02}-c_{01}} = 1 - \\ -\frac{4\nu}{\pi}\rho^{\nu}\left\{\sum_{j=1}^{2}\sum_{n=0}^{\infty}\int_{\eta_{jn}}^{(2n+1)\pi/2}\frac{\cos(\eta X)\Psi_j\eta d\eta}{\cos\eta} - \int_0^{\xi_0}\frac{\text{ch}(\xi X)\Psi_0\xi d\xi}{\text{ch}\xi}\right\} \tag{5.2}$$

$$\Psi_s = [Y_\nu(\Delta_s\rho)J_{\nu+1}(\Delta_s) - J_\nu(\Delta_s\rho)Y_{\nu+1}(\Delta_s)] \times \\ \times \exp(-\mu_s\text{Fo})/\{\Delta_s\mu_s\kappa_s[J^2_{\nu+1}(\Delta_s)+Y^2_{\nu+1}(\Delta_s)]$$

The values of the constants and variables are given in Secs 2 and 4. When $t-\infty(p\to 0)$ and $r\to R$, we have $\kappa R\approx\sqrt{(pD_r^{-1})}R$ and we can rewrite (5.1) in the form [2]

$$C_1(\rho,p)/(c_{02}-c_{01}) \simeq 2\nu\rho^{\nu-1/2}\exp[-\sqrt{pD_r^{-1}}\,R(\rho-1)/(R\sqrt{p^3D_r^{-1}}\,) \tag{5.3}$$

The function corresponding to this transform is given by

$$[c(\rho,\text{Fo})-c_{01}]/(c_{02}-c_{01}) \simeq 4\nu\rho^{\nu-1/2}\sqrt{\text{Fo}_1}\,i\text{erfc}[(\rho-1)/(2\sqrt{\text{Fo}_1}\,)] \\ \text{Fo}_1 = D_r t/R^2 \tag{5.4}$$

For $t\to\infty(p\to 0)$, subject to the assumptions adopted to obtain (4.8), we can rewrite (5.1) as follows:

$$U_1(x,r,p)/(c_{02}-c_{01}) \simeq [p(1+B_1p)]^{-1} \\ B_1 = a^2 D_x^{-1} T_1 \\ T_1 = m[1+A(1+K)][\nu(\rho^2-1)+1]/[2\epsilon\nu(\nu-1)] + \\ +(1+K)(1-X^2)/2 \tag{5.5}$$

The original function corresponding to this transform is

$$[u(X, \rho, \mathrm{Fo}) - c_{01}]/(c_{02} - c_{01}) \approx 1 - \exp(-\mathrm{Fo}/T_1) \tag{5.6}$$

As has already been mentioned, sorption occurs if $c_{02} > c_{01}$ and desorption occurs if $c_{02} < c_{01}$. The above exact and approximate solutions can be applied to compute the combined convective and molecular mass transfer in biporous media complicated by sorption (desorption) or ion exchange.

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